heat loss much less than coupling. It does, as noted in Section V, affect the overall signal loss, which is $\frac{1}{2}\exp[-(\alpha_0+\alpha_1)z]$, the signal sharing the heat losses of the two modes for $S_0z \gg 1$ [1].

This work may be extended to more forward spurious modes (but not backward spurious modes), and to systems whose output is a combination of several modes.

We do not know if the present limitations on impulse-response duration and bandwidth are fundamental or whether equalization might improve things. The latter possibility is suggested by the above discussion in connection with (70) and (71); for a given (large) coupling, the significant terms will tend to cluster around a specific value of n, and hence equalization might tend to help, although it could obviously never be perfect.

A single value for the spectral density S_0^{opt} of the coupling coefficient (or D_0^{opt} of the geometric imperfection) yields the shortest impulse response (and widest useful bandwidth) for all line lengths. This result was unforeseen.

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Simplified Theory for Post Coupling Gunn Diodes to Waveguide

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***Abstract*—There is a constant need for diode circuits employing rectangular waveguide. Coupling of the diode to the guide by using an inductive post is a popular method. The microwave circuit analysis of the equivalent circuit has been explored by complete theoretical analyses in the literature, but the results have been sufficiently difficult to apply that, in practice, recourse is often made to empiric characterization. This paper derives a simplified equivalent circuit based on a small perturbation approximation. The method is verified by experiment and is then used to evaluate a practical Gunn oscillator cavity.**

I. Introduction

COUPLING DIODES to rectangular waveguides is a frequent requirement in the design of microwave oscillators, detectors, and control devices. In effect, a mode transducer is required to couple the diode, which is essentially a lumped element with dimensions small compared to a wavelength, to the dominant TE_{10} mode whose fields are not nearly as concentrated in space as those surrounding the diode.

High Q resonators are very practical in waveguide, and this is advantageous with bulk-effect diodes whose operating frequency must be stabilized. Post coupling

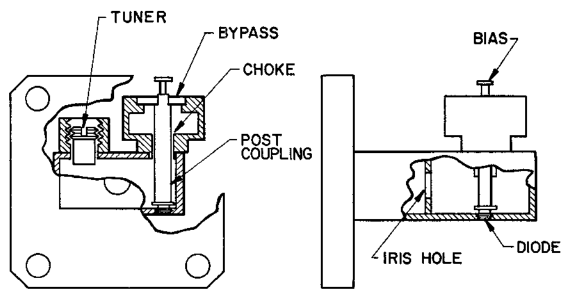

Fig. 1. Oscillator plan view.

the diode to the guide, as shown in Fig. 1, is common. The problem for the designer is how to estimate what load impedance will be experienced by the diode in this network.

The equivalent circuit that we will use is shown in Fig. 2. The reactance X_g associated with the gap will be neglected, and the conditions under which this approximation is valid will be discussed. The normalized[1] impedance $\bar{Z}_{ss'}$ can be calculated using transmission line theory and published [1]–[3] or measured values for the

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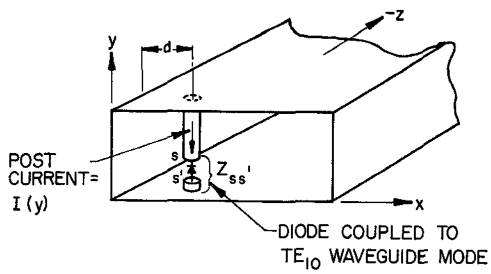
[1] In this paper, a bar over a variable denotes that its value is normalized to Z_g, the waveguide impedance.



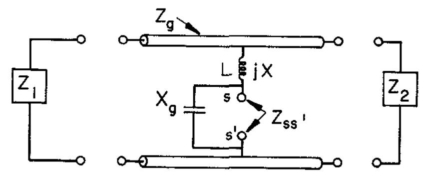


Fig. 2. (a) Schematic model of diode post coupled to waveguide. (b) Equivalent circuit looking out from diode. Waveguide has general terminations Z_1 and Z_2.

normalized impedances of the inductive post, iris, and screw tuner, which make up a practical resonator. The problem then is to determine a value for the waveguide impedance Z_g in order that the absolute impedance $Z_{ss'}$ is specified unambiguously [4]. This question is addressed in the literature via three basic approaches.

1) An experimental model is made and characterized by measurement [5]. This method is direct but time consuming, and the results may not be of sufficient generality to preclude the need for repeated measurements of subsequent device designs.

2) A theoretical analysis is made based upon the coupling of the RF current on the post to the infinite set of TE_{mn} waveguide modes [6]–[8] or, more completely, to the TE_{mn} and TM_{mn} mode sets [9]. Such solutions have the advantage of analyticity, but their use generally requires calculations that are impractical to perform without a computer, and physical insight is easily lost.

3) The equivalent circuit shown in Fig. 2 is applied and Z_g is defined[2] according to the voltage that, for a given power, would have existed at the post position in the waveguide were the post not present [10]. This approach is analytic, but such an *a priori* assumption for Z_g seems physically unsatisfactory since with an uninterrupted post in place the tangential field at the post, and hence this voltage, is identically equal to zero.

The objectives of this paper are to show the following.

1) The "voltage-power" definition for Z_g can be derived rigorously and simply using an analysis based on the coupling of the current on the post to the dominant TE_{10} mode.

2) The simplified equivalent circuit shown in Fig. 2 frequently can be used with X_g neglected, since the impedance referred to the gap in oscillator designs is often much smaller than X_g.

3) Practical Gunn diodes will operate under the low impedance conditions for which this simplified analysis method applies.

II. Theory

A. Validity Discussion

The waveguide impedance shall be defined as that characteristic impedance Z_g which produces for a given incident power the same current in the inductor of the equivalent circuit as exists on the uninterrupted waveguide post (i.e., with gap terminals ss' shorted). It follows that the resulting equivalent circuit approaches an exact model as the fields in the vicinity of terminals ss' approach the field distribution that surrounds the post before the section is removed.

If a small impedance is installed in the gap, the current on the post and the magnetic field surrounding it are little changed. Accordingly, the coupling to the TE_{m0} mode set and the corresponding equivalent inductance of the post are approximately the same. However, the finite voltage that can now exist at ss' excites TE_{mn} modes, for which generally neither m nor n is zero. It is to take into account this added coupling to evanescent modes that X_g appears in the equivalent circuit.[3] To include this coupling makes the analysis much more complex since the voltage at the gap is not known beforehand, whereas the current on an uninterrupted post is relatively easy to calculate. This is the motivation for the simplified analysis method.

It cannot be said that X_g is so large in all cases that its parallel effect on the driving-point impedance at ss' can be neglected without further consideration.

For example, with the gap at the bottom of the post, X_g approaches zero as the post height approaches one-half wavelength.[4] This can be appreciated by noting that, from the gap terminals, the post appears as a TEM transmission line that is short-circuit terminated at the opposite waveguide wall. Thus a short-circuit input impedance is to be expected. In this case, the value for X_g could be increased by centering the gap along the y dimension of the post. Moreover, this short-circuit resonance might be used to advantage in some device designs. However, the example given is an extreme; waveguides used in the dominant mode have b dimensions sufficiently less than a half wavelength that such low values for X_g are not usually encountered.

[2] This "voltage-power" definition for Z_g was also found useful in matching resistive films to waveguide. (See L. Lewin, "A physical interpretation of impedance for rectangular waveguide," *Proc. Phys. Soc. London*, pp. 868–869, May 1, 1958.)

[3] In this discussion, the parallel plate capacitance created at the post gap is not considered to be included in X_g, but rather is to be included within the "impedance that is installed within the gap."

[4] R. L. Eisenhart demonstrates this fact by showing that the net post reactance becomes independent of the gap width (and accordingly the capacitance of the gap) when the waveguide b dimension is one-half wavelength. [See [9], Fig. 22(a).]

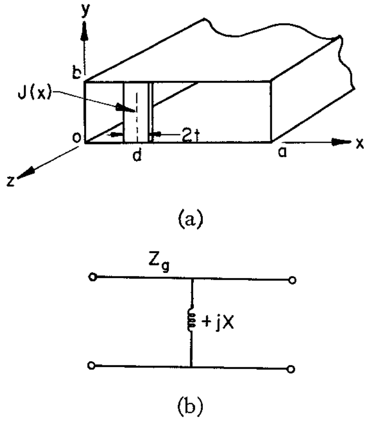


Fig. 3. (a) Thin (in z direction) inductive strip obstacle. (b) Equivalent circuit of uninterrupted post.

It is likely that a useful lower bound for X_g usually can be made by estimating the minimum characteristic impedance of this TEM mode. Then, taking into account the electrical length of the post from the gap to the waveguide broadwall, a minimum value for X_g can be estimated. Unless the gap is located at the end of the post, two such reactances must be calculated (looking from the gap toward each waveguide broadwall) and their sum used as an estimate of X_g. Alternatively, since the analysis is to be used to design a Gunn resonator, the circuit resonant frequency with the gap shorted can be compared with the oscillating frequency obtained with the Gunn diode installed. If both frequencies are nearly the same, the neglect of X_g is justified and the computation of the load impedance seen by the diode can be carried out using the simplified approach presented in this paper.

B. Derivation of the Waveguide Absolute Impedance

Consider the inductive strip shown in Fig. 3 having width $2t$, negligible thickness, and located a distance d from the narrow waveguide wall. The thin-strip rather than post geometry is used as a simplifying approximation so that in the analysis the current on the inductive obstacle is all contained in the $z=0$ plane. This causes no loss of generality, since we assume that the post cross-sectional dimensions are small insofar as the post current distribution is concerned. In order to establish Z_g, it is necessary to relate the power propagated by the TE_{10} wave that is radiated in both directions down the waveguide by a current I on the post. For the calculation, let the current on the strip be induced by a TE_{10} mode wave, defined by (1) and traveling in the $-z$ direction, and solve for the scattered (reflected) TE_{10} wave using the waveguide Green's function:[5]

$$E_1 = E_0 \sin(\pi x/a) \cdot e^{-\Gamma_1 z} \tag{1}$$

where $\Gamma_n = \sqrt{((n\pi)/(a))^2 - ((2\pi)/(\lambda_0))^2}$ and the sinusoidal time dependence is assumed.

[5] See Collin [11] for a general treatment of waveguide Green's functions.

An infinite number of TE_{M0} waves are excited by the strip and their electric fields combine with the incident TE_{10} mode to give zero tangential electric field on the strip. All of the induced TE_{M0} waves except the TE_{10} are evanescent so long as the waveguide is operated in the dominant-mode frequency region. They affect the energy storage of the strip obstacle and accordingly its normalized reactance jX,[6] but only the scattered dominant-mode reflection need be evaluated in this analysis. This reflected wave magnitude is defined in (2a) and evaluated by (2b).

$$E_R = RE_0 \sin(\pi x/a) \cdot e^{\Gamma_1 z} \tag{2a}$$

$$E_R = \frac{-j\omega\mu}{a\Gamma_1} \int_{d-t}^{d+t} J(x) \cdot \sin(\pi x/a) \cdot dx \tag{2b}$$

where $R = E_R/E_0$ equals the complex reflection coefficient.

For a narrow strip $J(x) \approx J_0 = \text{constant}$ and $J_0 \cdot 2t \approx I$, the peak current induced on the strip. Integrating (2) gives

$$E_R = \frac{-j\omega\mu}{a\Gamma_1} \cdot \sin(\pi d/a) \cdot I. \tag{3}$$

Now the average incident propagating power P is related to the incident TE_{10} wave of peak amplitude E_0 by the following:

$$E_0 = \sqrt{\frac{2P}{ab} \cdot \frac{\lambda_g}{\lambda_0} \cdot \sqrt{\frac{\mu}{\epsilon}}}\,. \tag{4}$$

Equations (3) and (4) can be combined to give the following:

$$2R^2 P = \sqrt{\frac{\mu}{\epsilon}} \cdot \frac{\lambda_g}{\lambda_0} \cdot \frac{b}{a} \cdot \sin^2(\pi d/a) \cdot I^2. \tag{5}$$

Alternatively from circuit theory, a reactance producing the reflection coefficient R when connected across a Z_g line (see Fig. 4) has a peak current I passing through it which is related to the incident average power P according to the following:

$$2R^2 P = Z_g \cdot I^2. \tag{6}$$

Comparing (5) and (6), it follows that the appropriate definition for waveguide impedance that we seek is given in the following:[7]

$$Z_g = 2 \cdot \sqrt{\frac{\mu}{\epsilon}} \cdot \frac{\lambda_g}{\lambda_0} \cdot \frac{b}{a} \cdot \sin^2(\pi d/a). \tag{7}$$

[6] The algebraic sign of reactances and susceptances described in this paper is included in the symbol. Thus the susceptance of an inductive iris is written $+jB$ where the quantity B is negative.

[7] Frequently, the factor $\sin^2(\pi d/a)$ is separated from the definition of Z_g, and is represented as an ideal transformer whereby the coupling of the post to the waveguide is varied by its position d. This would be an unnecessary complication in this analysis since Z_g is only defined in terms of the post current.

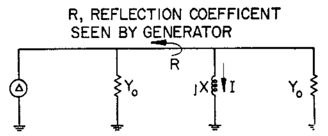


Fig. 4. Lumped circuit used to derive absolute impedance equivalent for the post obstacle in waveguide.

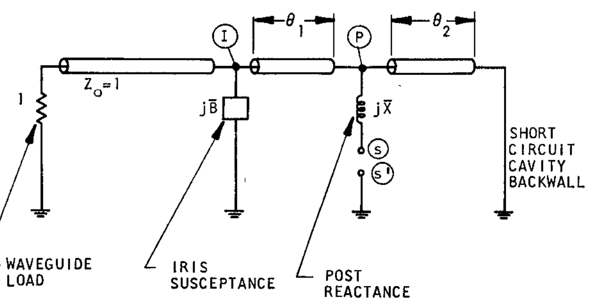


Fig. 5. Normalized equivalent circuit for a lossless cavity oscillator.

It is interesting to note that this Z_g definition, which is based on the post current and the TE_{10} mode power, is the same value that would be obtained were Z_g defined in terms of the TE_{10} voltage across the guide at d with no post present. This is explained heuristically by postulating that a virtual voltage exists at the post location which is just cancelled by the fields induced by the post current.

C. Use of the Equivalent Circuit

1) *Lossless Cavity Case:* With Z_g appropriately defined, the absolute equivalent circuit for the waveguide cavity resonator in Fig. 1 can be completely established. To do so, it is convenient to use a normalized circuit for analysis, since published or measured impedance data for posts, irises, and screw tuners are in normalized form. The calculated value for $\overline{Z}_{ss'}$ is then multiplied by Z_g to determine the absolute load impedance seen by a packaged diode installed in series with the post. The equivalent circuit for a cavity resonator is shown in Fig. 5. For simplicity, the cavity is assumed to be lossless. Losses will be taken into account as a perturbation later.

If the post is located an electrical length θ_1 from an iris having susceptance $j\overline{B}$, the real part of the normalized admittance $\overline{Y}$ of the iris and a matched waveguide load, transformed to the post position P, is given by the following:

$$\text{Re}\,(\overline{Y}) = \frac{1 + \tan^2\theta_1}{(1 - \overline{B}\tan\theta_1)^2 + \tan^2\theta_1}. \tag{8}$$

Since the cavity is presumed lossless so far, this must be the transformed admittance of the load at P. A further transformation to the device terminals ss' is effected by the post reactance $j\overline{X}$. This transformation factor is obtained by converting the admittance consisting of Re $(\overline{Y})$ and the susceptance of the post to an equivalent series impedance. Since this is a resonator circuit, the Q of this admittance $(\overline{X}\cdot\text{Re}\,(\overline{Y}))^{-1}$ should be at least 10, and, therefore, the ratio between resistance values in the parallel and series equivalent circuits can be approximated by the factor Q^2 with an error of less than one percent.

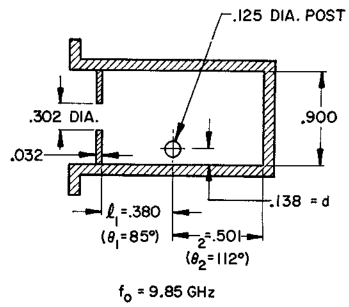


Fig. 6. Plan view of oscillator constructed in WR-90 waveguide ($a = 0.900$ in, $b = 0.400$ in) used to verify impedance calculations.

Using this approximation and multiplying by Z_g to get absolute impedance, the value of series resistance at ss' representing the waveguide load is given by the following:

$$R_{ss'} = \frac{Z_g\overline{X}^2(1 + \tan^2\theta_1)}{(1 - B\tan\theta_1)^2 + \tan^2\theta_1} \quad \text{(lossless cavity)} \tag{9a}$$

$$R_{ss'} \approx \frac{Z_g\overline{X}^2}{B^2}, \qquad \text{if } \theta_1 \approx \frac{\pi}{2} \text{ and } |B| \gg 1. \tag{9b}$$

Of course, at resonance the net susceptance at P must be zero. This is the tuning condition for the cavity. In practice, $\overline{B} \gg 1$ and the quarter-wave spacing transforms it to a small susceptance at P. Also, by assumption, the diode impedance is small compared with X. Therefore, the cavity resonates at about the frequency for which

$$-(1/\overline{X}) = \cot\theta_1 + \cot\theta_2. \tag{10}$$

2) *Cavity with Loss:* As a first-order approximation, cavity losses can be taken into account by multiplying the calculated value for R_{ss} by an appropriate factor. To determine the factor, it is assumed that R_{ss} is increased by cavity losses in direct proportion to the ratio of the total power dissipated in the cavity and waveguide load to that which is dissipated in the load. This ratio is easily written in terms of the unloaded and external-cavity quality factors Q_U and Q_E[8] given by (11). Thus the value for R_{ss} corrected for cavity losses becomes

$$R_{ss'} = \frac{Z_g\overline{X}^2(1 + \tan^2\theta_1)}{(1 - \overline{B}\tan\theta_1)^2 + \tan^2\theta_1}\cdot\left(\frac{Q_U + Q_E}{Q_U}\right). \tag{11}$$

III. Experimental Verification

Critical dimensions of a Gunn oscillator cavity used to verify the impedance calculations are shown in Fig. 6. A measured evaluation of $\overline{B}$ was selected because the value depends critically upon the thickness of the iris

[8] For definition and measurement of Q values see J. R. Altman, *Microwave Circuits.* New York: Van Nostrand, 1964, pp. 228–231.

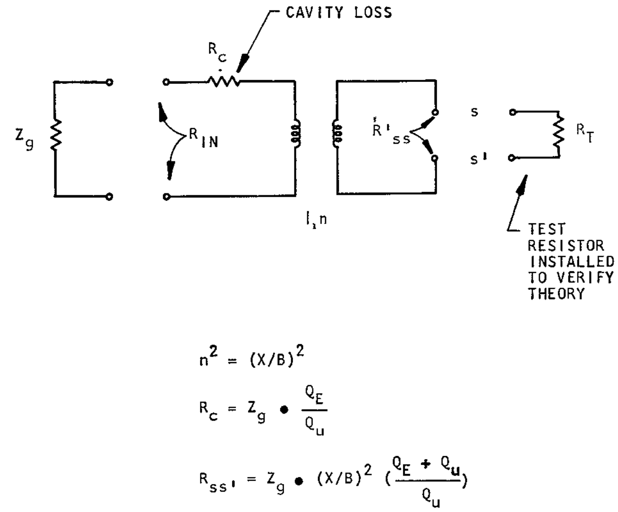


Fig. 7. Equivalent circuits showing transformation between terminals ss' and waveguide load Z_g at resonance.

plate near the opening. This can be difficult to assess if there are burrs or chamfers present near the hole. Similarly, the experimental value for $\overline{X}$ was used since, in practice, $\overline{X}$ is affected somewhat by the reactance of the RF choke used to introduce diode bias. With the post gap terminals shorted together, the cavity is resonated at 9.85 GHz with unloaded and external Q values of $Q_U = 1680$ and $Q_E = 652$. The magnitude of $\overline{B}$ was estimated, using (12)[9] and an approximate value $\overline{X} = 2$, to be 17.2:

$$Q_E = \frac{\overline{B}^2}{2} (\theta_1 \cdot \csc^2 \theta_1 + \theta_2 \csc^2 \theta_2 + 1/\overline{X}). \qquad (12)$$

Using this value for $\overline{B}$ together with the electrical lengths θ_1 and θ_2 at resonance, it is found with a Smith chart that the post presents a normalized susceptance $(1/\overline{X})$ of -0.39 at resonance, hence a more accurate value $\overline{X} = 2.56$ is determined. The equivalent circuit for the resonant cavity that couples the waveguide load to the diode terminals ss' is as shown in Fig. 7.

For the present case, the post separation from the sidewall and the waveguide dimensions results in $Z_g = 96.2\ \Omega$, which is coupled to the diode gap by the post and iris through the impedance transformation factor $(\overline{X}/\overline{B})^2 = 0.0221$. Were the cavity lossless, the value for load resistance at ss' would be 2.12 Ω. Losses increase this value by the factor 1.39, as determined by (11), and, hence, $R_{ss'} = 2.95\ \Omega$.

One method of verifying the theory is to install a known resistance R_T at terminals ss' and to measure the resistance R_{in} seen by looking into the cavity at the waveguide load port. To do this, a batch of homogeneously doped silicon resistors was made. The lowest resistance value available was 1.55 Ω (measured at dc). The resistor was in the form of a cylinder with metalized top and bottom contacts with a 0.014-in height and a 0.045-in diameter. The corresponding resistivity is 0.43 Ω·cm, and skin depth near 10 GHz is 0.010 in. The value of test resistance R_T is then 1.95 Ω at 10 GHz, taking skin effect into account [12].

The transformed resistance R_{in} depends upon both the transformation ratio $(\overline{X}/\overline{B})^2$ and the value of the cavity-loss resistor R_c. For example, if $Q_U = Q_E$, then $R_c = Z_g$ and a match is obtained just by shorting the terminals at ss'. In the experimental cavity, $R_c = 96.2\ \Omega \cdot (652/1680) = 37.4\ \Omega$. With the 1.95-Ω silicon resistor at ss', the net resistance at the input $R_{in} = 37.4 + (1.95) \cdot (1/0.0221) = 126\ \Omega$. Viewed from a waveguide with a 96.2 characteristic impedance, the VSWR should be 1.31. When this experiment was performed, the measured VSWR was found to be 1.54, verifying the theory within the experimental accuracy of the Q and VSWR measurements upon which the calculations were based.

When the resistive load was replaced by a Gunn diode, 14 mW of power was obtained at 9.92 GHz. The fact that the operating frequency was so close[10] to the resonant frequency with the post gap shorted reinforces the assumption that the gap reactance X_g can be neglected.

IV. Gunn-Diode Operation

A. Resistive Loading

The utility of the simplified impedance calculation can be seen from a practical example of a Gunn-diode circuit analysis. An oscillator design in which the post was located 3 λ/8 from the iris was found to yield a wider range of power adjustment. The oscillator is as shown in Fig. 6 but with $l_1 = 0.750$ in, $l_2 = 0.350$ in, and $d = 0.100$ in. Resonance with the gap shorted is at 8.8 GHz, at which $\overline{X} = 2.0$ and $Z_g = 59\ \Omega$.

The oscillator was tested with successively enlarged iris hole diameters. The power output of the oscillator is shown in Fig. 8 plotted as a function of the calculated impedance load seen at the gap in the post. The post reactance was determined from the resonant frequency shift of the cavity. Iris susceptance was estimated from the hole size, and cavity losses were neglected. The resulting calculated load resistance values are approximate, but they show the relatively wide impedance range, more than 10 to 1 over which the Gunn diode can develop useful power.

The effects of iris thickness are seen from the two data points obtained with 0.290-in iris holes. A factor of two

[9] This expression is derived in the Appendix.

[10] As a measure of how much change in cavity resonant frequency is significant, note that with a normalized post reactance of $+j2$ located near the cavity center (on the z axis of the cavity), about a 5-percent or 500-MHz shift in cavity resonance occurs (the empty cavity resonance being about 500 MHz lower than with the post loading). If the post current distribution or magnitude were changed appreciably, the stored energy about the post and the equivalent post reactance would also change. If the resonance change is small compared to 500 MHz, the constant current assumption would seem to be reasonable.

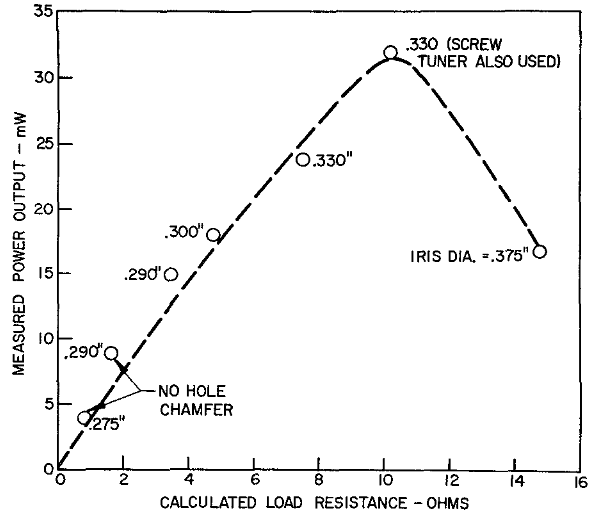


Fig. 8. Measured output power of a Gunn oscillator versus calculated load resistance.

increase in power was achieved merely by chamfering the hole in the 0.032-in iris so that a knife edge was formed at the 0.290-in diameter. Thick-iris susceptance calculations were used to determine the susceptance ($\overline{B}=-21$) before chamfering [9] and thin-iris data [5] were used to approximate the chamfered hole susceptance ($\overline{B}=-11$). Thereafter, as the iris aperture was enlarged, chamfered holes were used because it was felt the uncertainty caused by burrs could be minimized. From the data in Fig. 8, it can be seen that the last iris enlargement from a 0.330- to 0.375-in diameter increased the load resistance too much and the 30-mW power output obtainable with this diode when a slide screw tuner was used in front of the cavity was not achieved, emphasizing the sensitivity of iris susceptance and, accordingly, the diode-impedance loading with diameter.

B. Reactive Loading and Package Parasitics

The external diode-package parasitic inductance L_{ext} can be associated with the post inductance. While L_{int} is shunted by diode-package capacity C_p, its reactance magnitude at frequencies up to X band is considerably smaller than that of C_p, and so the package transformation effects in $R_{ss'}$ are negligible so long as the diode is used in a mode for which its Thevenin source impedance $Z_D=R_D+jX_{D'}$ is no more than a few ohms in magnitude. The diode-package parasitics can shift the cavity resonance, however, and this will now be examined.

Oscillation occurs when the load impedance $Z_{ss'}=R_{ss'}+jX_{ss'}$ presented to the packaged Gunn diode at the gap in the post is the negative of the Thevenin equivalent impedance Z_D of the packaged diode. Up to this point only the real component $Z_{ss'}$ has been determined. Near resonance, the load impedance $Z_{ss'}$ behaves as a series LRC loop resonant at f_0. Using network theory, the net reactance incurred for a small change Δf in frequency is related to R and the loaded Q by

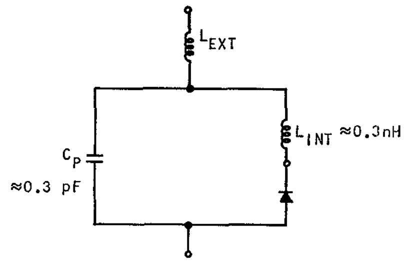


Fig. 9. Equivalent circuit for MA OD-111 pill diode package.

$$X_{ss'} = 2\left(\frac{\Delta f}{f_0}\right) Q_L \cdot R_{ss'}. \qquad (13)$$

The approximate package reactances for a pill package (MA package OD-111) are shown in Fig. 9. The height of the ceramic insulator of the package is 0.034 in. If the impedance magnitude of the Gunn-diode wafer is no more than a few ohms, the equivalent series reactance of the diode-package inductance and capacitance is about $+j15\ \Omega$ near 10 GHz. The fraction of the post reactance $\overline{X}Z_g \approx j100\ \Omega$, associated with the ceramic height, is (0.034 in/0.400 in)$\cdot \overline{X}Z_g$, or about $+j8\ \Omega$. Thus installation of the diode necessitates a shift in the cavity resonance frequency compared with the resonant frequency measured before the gap is cut in the post. Assuming $R_{ss'} \approx 1\ \Omega$ and $Q_L > 350$, (13) shows that the resonant frequency changes are less than one percent, and thus to a first-order approximation, package effects can be neglected when R_{ss} is this small. For larger values of $R_{ss'}$ the impedance transformation effected by the package reactances can be calculated using the equivalent circuits given, subject of course to the requirement that $R_{ss'}$ be small compared with the post reactance magnitudes X and X_g.

V. Conclusions

A rectangular waveguide operated in the dominant-mode frequency range induces a current uniform along the length of a post connecting its broad walls. The normalized post reactance is readily obtained from the literature or by measurement. The current magnitude can be derived compactly from the first term of the waveguide Green's function, and, from these two values, the "voltage-power" waveguide impedance definition is shown to apply.

Thereafter, if an impedance is installed in series with the post with small enough magnitude, the same absolute impedance waveguide equivalent circuit prevails, and a simple circuit analysis is possible.

The experiments described with Gunn diodes show that at X band, this low-impedance assumption is applicable and, therefore, the analysis should be valuable for designing such oscillators. The simplified analysis approximations described also should be useful with many other circuits wherein post transducers in waveguides are used. For example, since the post both supports a TEM mode as well as couples to the waveguide dominant mode, the TEM mode might be used as an idler circuit for a multiplier diode whose output power is then coupled to a resonant waveguide cavity by the post.

Appendix

The external Q of a cavity Q_E depends upon how the cavity is coupled to the load. For a cavity having an iris susceptance $\overline{B}$, the external Q is related to $\overline{B}$ as follows. Referring to Fig. 5, the resonance of a waveguide cavity having iris coupling and an inductive post $\overline{X}$ occurs when the net admittance at all planes along the propagation direction is zero. For convenience, consider plane P of the post.

If cavity losses are neglected and it is assumed that $\theta_1 \approx \pi/2$ and that $|\overline{B}| \gg 1$, the admittance at location P is given by the following:

$$\overline{Y}_P = \frac{1}{\overline{B}^2} - j\left(\cot\theta_1 + \cot\theta_2 + \frac{1}{\overline{X}}\right). \quad \text{(A1)}$$

Resonance $f = f_0$ occurs when $\text{Im}\,(\overline{X}_P) = 0$. The derivative of $\text{Im}\,(\overline{Y}_P)$, with respect to frequency f, evaluated at resonance, is given by the following:

$$\partial\left(\frac{\text{Im}\,(\overline{Y})}{\partial f}\right)\Bigg|_{f_0} = \frac{1}{f_0}\cdot\left(\theta_1 \csc^2\theta_1 + \theta_2 \csc^2\theta_2 + \frac{1}{\overline{X}}\right). \quad \text{(A2)}$$

By comparison, the derivative of the susceptance of a lossless lumped parallel LC circuit resonant at f_0 and having admittance

$$Y_L = G_{\text{EXT}} + j(\omega C - 1/\omega L) \quad \text{(A3)}$$

is

$$\frac{\partial\,\text{Im}\,(\overline{Y}_L)}{\partial f}\Bigg|_{f_0} = \frac{2}{f_0}\cdot 2\pi f_0 C \quad \text{(A4)}$$

and its external quality factor Q_E is

$$Q_E = \frac{2\pi f_0 C}{G_{\text{EXT}}} \quad \text{(A5)}$$

where G_{EXT} is the external load conductance coupled in shunt with the LC resonator. Comparing (A1) and (A3) and (A2) and (A4), the equivalences in (A6) and (A7) are found to apply near the resonant frequency of the iris-coupled cavity and its lumped-circuit equivalent.

$$G_{\text{EXT}} = \frac{1}{\overline{B}^2}\cdot \quad \text{(A6)}$$

$$2\pi f_0 C = \frac{1}{2}\left(\theta_1 \csc^2\theta_1 + \theta_2 \csc^2\theta_2 + \frac{1}{\overline{X}}\right). \quad \text{(A7)}$$

Substituting these values into (A5), the external Q for the cavity is obtained as (12) in the text, which was to be shown.

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